This write-up discusses the second case which consists of Phase 1 experiment with randomised block design (RBD) and Phase 2 experiment is also a randomised block design (RBD). Few different designs, which generated from each of two sets of design parameters, have presented.

Record the objective function for finding the optimal designs for this case can be written as

where and denote the average efficiency factors of animals and treatments in the Within Cages and Runs stratum, respectively, the term allows the DF of treatment in the Between Animals Within Cages and Runs stratum to be maximised. The term is to maximise the residual DF for conducting the test of the treatment effects. The weights for average efficiency factors of animals and treatment, terms maximising the DF associated with the treatment effects and residual mean squares are 5/9, 1/9, 2/9 and 1/9, respectively.

**The first experiment example consists of and .**

The allocations of cages, animals and treatments are presented here in matrices. The rows and columns are corresponding to the runs and tags, respectively. The cages are denoted by the upper case letters, the animals within the cages are denoted by the numeric numbers and the treatments are denoted by the lower case letters.

For the design of the first phase experiment, two animals were groups in each of three cages, Animal 1 of each cage are applied by Treatment “a” and Animal 2 of each cage are applied by Treatment “b”.

The theoretical ANOVA table for the Phase 1 experiment can be written as follows

$ANOVA

DF e Cag:Ani Cag

Between Cag 2 1 2 4

Between Cag:Ani

Trt 1 1 2 0

Residual 2 1 2 0

Within 6 1 0 0

$EF

Trt eff.Trt

Between Cag

Between Cag:Ani

Trt 6 1

Within

The aim of the allocation is to have as much of information in the Between Animals Within Cages stratum of the first phase experiment in the Within Runs stratum.

Using the objective function described above, the allocation of cages and animals can be shown as follows

[,1] [,2] [,3] [,4]

[1,] "B2" "A1" "B1" "A2"

[2,] "A1" "B2" "A2" "B1"

[3,] "C2" "C2" "C1" "C1"

The cages are orthogonal to tags, since each tag contains all three cages. However, the cages are confounded with the runs, because Runs 1 and 2 contain Cages A and B and Run 3 contain Cage C. As for the animals within cages, the confounding also occurred in the contrast of between Run 1 & 2 versus Run 3. In addition, the animals within cages are also confounded with the tag contrast of 1 & 2 versus 3 & 4. The treatment allocation is then shown as follows,

[,1] [,2] [,3] [,4]

[1,] "b" "a" "a" "b"

[2,] "a" "b" "b" "a"

[3,] "b" "b" "a" "a"

where the treatment effects is shown to be orthogonal to runs but confounded with the tags.

The theoretical ANOVA table of this design can be written as

$ANOVA

DF e Cag:Ani Cag Run

Between Run

Between Cag 1 1 2 4 4

Residual 1 1 0 0 4

Within

Between Cag 1 1 2 4 0

Between Cag:Ani

Tag 1 1 2 0 0

Trt 1 1 2 0 0

Residual 1 1 2 0 0

Residual

Tag 2 1 0 0 0

Residual 3 1 0 0 0

$EF

Tag Trt eff.Tag eff.Trt

Between Run

Between Cag

Residual

Within

Between Cag

Between Cag:Ani

Tag 3 2/3 1 1/9

Trt 16/3 8/9

Residual

Tag 3 1

where the random effects table shows the test for the treatment effects can be conducted. The fixed effects table shows the amount of treatment information been tested is 8/9. Compare to the theoretical ANOVA table of the Phase 1 experiment, one DF associated with residual MS in the Between Animal within Cages and Runs stratum is lost due to the confounding with the tag effects.

The next two designs were generated manually in attempt to improve the design.

The allocation of cage of second design is purposely to be completely confounded with the runs and the allocation of the animals within cages stays the same. The allocation of cages and animals can be shown as follows

[,1] [,2] [,3] [,4]

[1,] "A2" "A1" "A1" "A2"

[2,] "B1" "B2" "B2" "B1"

[3,] "C2" "C2" "C1" "C1"

where cages are confounded with the runs but orthogonal to the tags. The animals within cages are confounded with runs, but this confounding is already consider in the confounding between cages and runs. The animals within cages are confounded with the 2 contrast of Tag 1 & 4 versus 2 & 3 and Tag 1 & 2 versus 3 & 4.

The treatment allocation is the same as before

[,1] [,2] [,3] [,4]

[1,] "b" "a" "a" "b"

[2,] "a" "b" "b" "a"

[3,] "b" "b" "a" "a"

The theoretical ANOVA table of this design can be written as

$ANOVA

DF e Cag:Ani Cag Run

Between Run

Between Cag 2 1 2 4 4

Within

Between Cag:Ani

Tag 2 1 2 0 0

Trt 1 1 2 0 0

Residual

Tag 3 1 0 0 0

Residual 3 1 0 0 0

$EF

Tag Trt eff.Tag eff.Trt

Between Run

Between Cag

Within

Between Cag:Ani

Tag 4/3 2 4/9 1/3

Trt 4 2/3

Residual

Tag 18/11 6/11

where the random effects table shows the test for the treatment effects cannot be conducted, this is because of confounding of two tag contrasts to the animals within cages, 2 DF associated with the tag effects are in the Between Animal Within Cages and Runs stratum . In addition, the fixed effects table for this design shows the amount of treatment information been tested becomes 2/3. Hence, this design cannot be used.

The structure of second manually generated design is that the same cages are assigned in the 2-by-2 section of the runs and tags and the allocation of animals within cages stays the same. The allocation of cages and animals can be shown as follows

[,1] [,2] [,3] [,4]

[1,] "A2" "A1" "B1" "B2"

[2,] "A1" "A2" "B2" "B1"

[3,] "C2" "C2" "C1" "C1"

The cages are confounded with the one run contrast of Run 1 & 2 versus Run 3 and one Tag contrast of Tag 1 & 2 versus Tag 3 & 4. The confounding patterns of the animals within cages with runs and tags are same of the confounding of the cages with runs and tags.

The treatment allocation is the same as before

[,1] [,2] [,3] [,4]

[1,] "b" "a" "a" "b"

[2,] "a" "b" "b" "a"

[3,] "b" "b" "a" "a"

The theoretical ANOVA table of this design can be written as

$ANOVA

DF e Cag:Ani Cag Run

Between Run

Between Cag 1 1 2 4 4

Residual 1 1 0 0 4

Within

Between Cag

Tag 1 1 2 4 0

Between Cag:Ani

Tag 1 1 2 0 0

Trt 1 1 2 0 0

Residual 1 1 2 0 0

Residual

Tag 2 1 0 0 0

Residual 3 1 0 0 0

$EF

Tag Trt eff.Tag eff.Trt

Between Run

Between Cag

Residual

Within

Between Cag

Tag 2 2/3

Between Cag:Ani

Tag 1 2 1/3 1/3

Trt 4 2/3

Residual

Tag 3 1

where the random effects table shows the test for the treatment effects can still be conducted. The fixed effects table for this design shows the amount of treatment information been tested becomes 2/3. Therefore, it shows that the first design cannot be improved any further.

**The second example consists of and .**

The previous experiment example consists of the odd numbers of runs and cages. This example contains the even numbers of runs and cages.

For the design of this first phase experiment, two animals were groups in each of four cages, Animal 1 of each cage are applied by Treatment “a” and Animal 2 of each cage are applied by Treatment “b”.

The theoretical ANOVA table for the Phase 1 experiment can be written as follows

$ANOVA

DF e Cag:Ani Cag

Between Cag 3 1 2 4

Between Cag:Ani

Trt 1 1 2 0

Residual 3 1 2 0

Within 8 1 0 0

$EF

Trt eff.Trt

Between Cag

Between Cag:Ani

Trt 8 1

Within

Using the objective function described above, the allocation of cages and animals can be shown as follows

[,1] [,2] [,3] [,4]

[1,] "A1" "A2" "B1" "B2"

[2,] "A2" "A1" "B2" "B1"

[3,] "C1" "C2" "D1" "D2"

[4,] "C2" "C1" "D2" "D1"

For this case, the best allocation of cages has the same structure as the previous example where same cages are assigned in a section of 2 runs and 2 tags. Hence, both the cages and animals within cages are confounded with one run contrast of Run 1,2 versus 3,4 and one tag contrast of Tag 1,2 versus 3,4.

The treatment allocation can be shown as follows

[,1] [,2] [,3] [,4]

[1,] "a" "b" "a" "b"

[2,] "b" "a" "b" "a"

[3,] "a" "b" "a" "b"

[4,] "b" "a" "b" "a"

where the treatments are orthogonal to both runs and tags, because each run and tag contain 2 of each treatments.

The theoretical ANOVA table for the Phase 2 experiment can be written as follows

$ANOVA

DF e Cag:Ani Cag Run

Between Run

Between Cag 1 1 2 4 4

Residual 2 1 0 0 4

Within

Between Cag

Tag 1 1 2 4 0

Residual 1 1 2 4 0

Between Cag:Ani

Trt 1 1 2 0 0

Residual 3 1 2 0 0

Residual

Tag 2 1 0 0 0

Residual 4 1 0 0 0

$EF

Tag Trt eff.Tag eff.Trt

Between Run

Between Cag

Residual

Within

Between Cag

Tag 4 1

Between Cag:Ani

Trt 8 1

Residual

Tag 4 1

The random effects table shows the test for the treatment effects can be conducted with the same precision as the Phase 1 experiment; this is because the DF associated with the residual MS is still 3. In addition, 100% of the treatment information stays intact in the Between Animals Within Cages and Runs stratum for the test of the treatment effects.

The next manually generated design follows the structure of the best design generated from the previous experiment example, which is that two of four cages are assigned in a section of two runs and two tags. This allocation of cages and animals is shown as follows,

[,1] [,2] [,3] [,4]

[1,] "A1" "B2" "C1" "D2"

[2,] "B2" "A1" "D2" "C1"

[3,] "D1" "C2" "B1" "A2"

[4,] "C2" "D1" "A2" "B1"

For this allocation, the cages are orthogonal with both runs and tags. However, the animals within cages are still confounded with one run contrast of Run 1,2 versus 3,4 and one tag contrast of Tag 1,2 versus 3,4.

The treatment allocation stay the same can be shown as follows

[,1] [,2] [,3] [,4]

[1,] "a" "b" "a" "b"

[2,] "b" "a" "b" "a"

[3,] "a" "b" "a" "b"

[4,] "b" "a" "b" "a"

The theoretical ANOVA table for the Phase 2 experiment can be written as follows

DF e Cag:Ani Cag Run

Between Run

Between Cag:Ani 1 1 2 0 4

Residual 2 1 0 0 4

Within

Between Cag 3 1 2 4 0

Between Cag:Ani

Tag 1 1 2 0 0

Trt 1 1 2 0 0

Residual 1 1 2 0 0

Residual

Tag 2 1 0 0 0

Residual 4 1 0 0 0

$EF

Tag Trt eff.Tag eff.Trt

Between Run

Between Cag:Ani

Residual

Within

Between Cag

Between Cag:Ani

Tag 4 1

Trt 8 1

Residual

Tag 4 1

From the random effects table, one DF associated with tag effect is in the Between Animals within Runs stratum and one DF associated with the animals with cages is in the Between Runs stratum. This is because the confounding of the animals within cages with runs tags. All three DF of cage is now in the Within Runs stratum. The test for the treatment effects can still be conducted, but there is only one DF associated with residual MS for conducting the test. Therefore, the first previous design is more preferable.